RAHULA COLLEGE , MATARA   Third Term Test - 2012

Combined Mathematics – Grade 12   Time – 03 Hours

Answer 06 questions only.

01. (a) Express $y = 2x^2 - 3x + 1$ in the form of $y = A(x + B)^2 + C$. Find the symmetrical axis , the minimum value , the points of intersection with x and y axes , of this curve. Sketch the curve.

(b) Let $y = ax^2 + bx + c$. The minimum value of this curve is $-1/4$ and the symmetrical axis is $x = 3/2$. If the curve goes through the point (1,0) , find the constants a ,b and c .

02. (a) $P(x) = x^3 - 3x^2 + 2x + 1$ $Q(x) = x^2 + 1$. Find the quotient and the remainder when P(x) is divided by Q(x) using the Long division method.

(b) $P(x) = ax^3 + bx^2 + c$. When P(x) is divided by $(x - 1)$, $(x + 1)$ and $(x - 2)$ separately , the remainders are 4 ,−2 and 22 respectively. Find the constants a , b and c .

(c) $P(x) = x^4 - 3x^2 + x + 5$. Using the Remainder theorem , find the remainder when P(x) is divided b by $(x^2 + 3x + 2)$

03. (a) (i) The function f is defined by $f: \mathbb{R} \to \mathbb{R}$ $f: x \to x^2 - 1$. State the range of f

(ii) If the function g is defined by $g(x) = \sqrt{f(x)}$, state the domain of g .

(b) Convert into partial fractions.

(i) $\dfrac{2x^2 - x + 1}{x(x-1)^2}$

(ii) Find the constants A,B C and D such that

$$\frac{2x^3 + 5x^2 - 2x - 2}{(x-1)(x-2)} = Ax + B + \frac{C}{(x-1)} + \frac{D}{(x+2)}$$

04. In the triangle ABC , A =(1,1) , B = (5,2) , C = (2,4) . D is the midpoint of AB. The straight line which is drawn parallel to BC through D , meets AC at the point E. The point F lies on AC such that AC = CF. G is the midpoint of BF.

(i) Find the coordinates of the points D , E and F.

(ii) Find the areas of the triangles ABC and ADE.

(iii) Find the coordinates of G and determine the area of the quadrilateral ABGC.

(iv) Find the coordinates of the centroid of the triangle ABC.

05. (a) Expand Sin (A + B) and Cos (A+ B) . Show that $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A . \tan B}$

Hence , express $\tan 2\theta$ in powers of $\tan\theta$.

Show that $\sin 2\theta = \dfrac{2\tan\theta}{1 + \tan^2\theta}$ and $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

If $\tan\theta = 4/3$, find the values of $\tan 2\theta$, $\sin 2\theta$ and $\cos 2\theta$

(b) Using the expansions of Sin (A – B) and Cos (A –B) , find the values of Sin 15^0 and Cos 15^0
Hence find the value of tan 15^0

06.(a) If Sin α + Sin β = a and Cos α + Cos β = b , prove that $\cos^2\frac{(\alpha-\beta)}{2} = \frac{a^2+b^2}{4}$

Hence , find the value of Cos (α – β).

(b) Sin A = $\sin^2$B and 2 $\cos^2$A = 3 $\cos^2$B where A and B are acute angles. Calculate the value of B.

(c) Find the general solution of the equation $\sin^3\theta - 2\cos^2\theta - 3\sin\theta + 2 = 0$

07. Two trains A and B travel along parallel tracks . At a certain instant the train A passes a point X with constant velocity of 50 ms^{-1}. 10 seconds later , the train B starts motion in the same direction from rest at a point Y which is 50 m ahead of X. The train B travels with uniform acceleration of 2 ms^{-2} until it reaches the velocity 70 ms^{-1} which it maintains afterwards. Draw the velocity – time graphs for A and B on the same diagram.
Hence , find the time taken by B to overtake A.

08. Define $\lambda \underline{a}$ when $\underline{a}$ is a non zero vector . λ is a scalar.

$\underline{a}$, $\underline{b}$ and $\underline{c}$ are non zero vectors no two of which are collinear. The vector $\underline{a} + 2\underline{b}$ is collinear with $\underline{c}$ and the vector $\underline{b} + 3\underline{c}$ is collinear with $\underline{a}$. Find a relation among $\underline{a}$, $\underline{b}$ and $\underline{c}$ in the form of $\alpha\underline{a} + \beta\underline{b} + \gamma\underline{c} = 0$. Hence evaluate $| \underline{a} + 2\underline{b} + 6\underline{c} |$.

09. Explain the Larne's Theorem.

A smooth ring P of mass *m* is free to slide on a smooth vertical circular wire. P is connected to a particle Q of mass *m* by a light inextensible string which goes over a smooth peg at the highest point of the circle. At the equilibrium position , OP makes an acute angle α with the vertical direction . O is the centre of the circle.
In the diagram , mark all the forces acting on the system.
Find the tension on the string , the reaction of the wire and determine the value of α.