Rahula College -Matara

Third Term Test – 2010

Combined Mathematics Grade 12 Time -03 Hours

Answer 10 questions of Part A and 05 questions of Part B

Part - A Compulsory

01. Expand Sin (A + B) and Cos (A + B)
   i. Express Sin 2θ in powers of Sin θ
   ii. Express Cos 2θ in powers of Cos θ

02.Sketch the graph of $f(x) = \sin x$. Show that f is a function.
State the Domain , Co domain and Range of f

03. Express 2 Sin 5x Cos 3x as the sum or difference of two linear trigonometric ratios.

04. An object is projected vertically upwards with a velocity of 40 ms^{-1} , from a point O on the earth. Taking acceleration due to gravity as 10 ms^{-2} , find
i the maximum height that the particle will rise.
ii the time taken for it.

05.A particle moving along a straight path , starts motion from rest at the point O and moves under uniform acceleration for time t_1. Subsequently ,it moves with uniform velocity for time t_2 and later takes time t_3 to come to rest at point P under uniform retardation .
After that, the particle moves towards O under uniform acceleration for time t_4 and subsequently with uniform velocity for time t_5 . And finally under uniform retardation for time t_6 and comes to rest at point O. Draw
(a) a velocity – time curve,
(b) a displacement – time curve , for the motion of the particle

06. Sketch the graph of $y = 2x^2 - 3x + 1$

07. $ax^2 + bx + c = 0$ $a \neq 0$, $a,b,c \in R$ .If it's roots are α and β , show that $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$

08. If A = (1,1) , B = (3 ,2) , C =(2 , 4) , find the area of the triangle ABC.

09.$\mathbf{i}$ and $\mathbf{j}$ are the unit vectors along the X and Y axes on a Cartesian plane. Find $\mathbf{r}_1$ and $\mathbf{r}_2$ the position vectors of the points (1,3) and (2 ,1) respectively , and find the value of $|\mathbf{r}_1 + \mathbf{r}_2|$

10. According to the usual notation , in the triangle ABC $a = 4$, $b = 5$ $c = 6$. Using the Cosine Rule, calculate the value of the angle C.

11. If $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ in the regular hexagon ABCDEF , find $\overrightarrow{CD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

12. What is meant by the angle between two vectors ?
Find the angle between the vectors $3\mathbf{i} + 8\mathbf{j}$ and $-2\mathbf{i} + 17\mathbf{j}$, using the Dot Product.

13. Write down the sets of (i) Natural numbers (ii) Integers (iii) Rational numbers (iv) Irrational numbers, out of the numbers given below

17 , -3 , 0 , 26.02 , 6.03×10^{23} , 2.8×10^{5} , 0.85 , 2 / 3 , -3 /4 , 5/3 , 22/ 7 , - 0.761 ,

$\sqrt{2}$, $\sqrt{7}$, $3\sqrt{25}$, $12\frac{2}{3}$, $24\frac{3}{5}$, π , e , $\log_{10} 2$, $\sin 53^{0}$

14. Find the rational number which is equivalent to $4.\dot{3}2\dot{1}$

15. If $\sqrt{3} = 1.73201$ and $\sqrt{2} = 1.41421$, find the value of $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{2-\sqrt{3}}$

16. (i) Which of the following equations represent functions.

$y = 2 - 3x$, $y = x^3$, $y^2 = x$, $|y| = -x$

(ii) Draw the graph of the function defined by , $f(x) = \begin{cases} 2 - x & , x < 2 \\ 2x - 1 & , x \geq 2 \end{cases}$

17.(i) Are the two functions $f(x) = x + 1$ and $g(x) = \frac{x^2 - 1}{x - 1}$ equal ? Give reasons .

(ii) If $f(x) = x^2$ and $g(x) = \frac{1}{1 - x}$, show that $g(x^2) = \frac{1}{1 - f(x)}$

What is the Domain of the function $g[f(x)]$

PART - B

01. (i) Express $y = x^2 - 3x + 2$ in the form $y = (Ax + B)^2 + C$. Here A , B & C constants which have to be determined. Hence, find the symmetrical axis and the minimum value of y . Also find the points of intersection with x and y axes , and sketch the graph of y.

(ii) If $P(x) = x^4 - 3x^2 + 2x + 1$ and $Q(x) = x + 2$, find the quotient and the remainder when P(x) is divided by $Q(x)$

(iii) $P(x) = x^4 - 3x^3 + x^2 + 3x - 2$.

(a) Show that $(x + 5)$ is not a factor of P(x)

(b) Show that $(x - 1)$ and $(x + 1)$ are the factors of P (x) , and find all the other factors of P (x)

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(a) Show that $(x + 5)$ is not a factor of P(x)

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02. (i) α and β are the roots of the quadratic equation $x^2 - px + q = 0$. Express the following relations in terms of p and q.

(a) $\alpha^2 + \beta^2$ (b) $\alpha^3 + \beta^3$

Find the equation whose roots are α^2 and β^2 in terms of p and q. **Hence**, deduce the quadratic equation whose roots are $(\alpha^2 + 2)$ and $(\beta^2 + 2)$.

(ii) If the quadratic equation $x^2 - (k + 2)x + 2k = 0$ has real and equal roots, find the value of k.

03. X and Y are two cars travelling in two horizontal parallel routes. At a certain moment, X car passes a location A at a uniform velocity of $U\ ms^{-1}$. t_0 time later, the Y car starts motion from rest and travels under uniform acceleration $a\ ms^{-2}$. The two cars will meet again T time after Y starts and at that moment the velocity of Y is $V\ ms^{-1}$.

Draw a velocity – time graph for motion until they meet again, And build up a relationship among U, a, t_0 and T.

If $U = 40\ ms^{-1}$, $a = 25/4\ ms^{-2}$, $t_0 = 4$ s, find the value of T. Also find the value of V.

04.(i) A motor car starts with initial velocity U, moves under uniform acceleration a and attains a velocity V after time t and the displacement covered is S. Using the definitions, derive the equations for motion.

(ii) The distance between a point O on the ground and a point B vertically above A, is 60 m. A particle is projected vertically upwards from A, and at the same moment another particle is freely dropped from B. How long will A and B take to meet each other ?

Find the height above the ground and the velocity of each particle at the moment they meet.

05. (i) Taking $\pi/3 + \pi/4 = 7\pi/12$, find the value of $\sin(7\pi/12)$. **Hence**, express the value of $\text{Cosec}\ (7\pi/12)$ in surd form.

(ii) Expand $\tan(A + B)$. Hence, express $\tan 2\theta$ and $\tan 3\theta$ in terms of $\tan\theta$ and it's powers.

(iii) If $A + B = \pi/3$ and $\cos A + \cos B = 1$, show that $\cos(A - B) = -1/3$

06. (i) If $\sin\alpha = 12/13$ $[0 < \alpha < \pi/2]$ and $\cos\beta = -3/5$ $[0 < \beta < 3\pi/2]$, find the value of $\sin(\alpha + \beta)$

(ii) If $A + B = \pi/4$, show that $(1 + \tan A)(1 + \tan B) = 2$

(iii) If $A + B + C = \pi$, show that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

07.(i) f is defined by $f : x \longrightarrow x^2 + 2$, $x \in R$

(a) Express the Range of f

(b) Show that f is a One to One function

(c) Find $f^{-1}(x)$ in terms of x

(ii) Find the set / sets of values which satisfy the following inequalities

(a) $(x-1)(x+1)(x-2) \geq 0$

(b) $\frac{(x-2)(x-4)}{(x-1)} \leq 0$

08. If the coordinates of two points A and B are (-2, 3) and (1, 5),

(i) Write down the position vectors of A and B

(ii) Find $\overrightarrow{AB}$

(iii) Find the magnitude of $\overrightarrow{AB}$

(iv) Find the unit vector in the direction $\overrightarrow{AB}$

[$\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the directions of the axes OX and OY respectively]

09. (i) Find the resultant of the system of coplanar forces $3\mathbf{i} + 8\mathbf{j}$, $7\mathbf{i} + 2\mathbf{j}$, $-8\mathbf{i} - 3\mathbf{j}$, $9\mathbf{i} - 9\mathbf{j}$, $-2\mathbf{i} + 17\mathbf{j}$

(ii) Using vectors, obtain the equation $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ where R is the resultant of the two forces P and Q.

(iii) At time $t = 0$, the position vectors of two objects P and Q with respect to an origin O are $-7\mathbf{i} + 5\mathbf{j}$ and $-3\mathbf{i} - 2\mathbf{j}$ respectively. They move with uniform velocities $3\mathbf{i} - \mathbf{j}$ and $2\mathbf{i} + \lambda\mathbf{j}$ respectively. If they meet each other after time T, find the value of λ and T.

10. A system of coplanar forces 3, $8\sqrt{3}$, 2, $3\sqrt{2}$, 3 and $2\sqrt{3}$ Newton act on a particle. These forces make angles 60^0, 150^0, 225^0, 270^0 and 300^0 with the first force respectively.
Find the magnitude and the direction of the resultant of the system.